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ANALYTICAL STUDY OF MHD FREE CONVECTIVE HEAT AND MASS TRANSFER FLOW BOUNDED BY AN INFINITE VERTICAL PLATE WITH THERMAL RADIATION AND CHEMICAL REACTION

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ABSTRACT

The objective of this paper is to study an unsteady Magneto hydrodynamic (MHD) viscous, incompressible free convective flow of an electrically-conducting, Newtonian fluid through porous medium bounded by an infinite vertical porous plate in the presence of radiation, chemical reaction, heat source and surface temperature oscillation and as well as concentration oscillation. The governing equations are solved analytically using complex variables. Solutions for the velocity field, the temperature field and concentration field are obtained and discussed through graphs. The expressions for the Skin friction, the rate of heat transfer in the form of Nusselt number and the rate of mass transfer in the form of Sherwood number are also derived and discussed. It is interesting to notice that Steady mean flow velocity is decreased with increasing of thermal radiation parameter, magnetic parameter, chemical reaction parameter, Heat source parameter and Schmitt number, while it is increased with the increase of Grash of number and modified Grash of number. Unsteady main flow velocity is decreased with increasing of thermal radiation, magnetic parameter, chemical reaction, Heat source parameter, frequency of oscillation and Schmitt number, while it is increased with the increase of Grash of number and modified Grash of number.

KEYWORDS: MHD, Free Convection, Thermal Radiation, Chemical Reaction, Porous Medium and Vertical Plate

INTRODUCTION

Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems etc. Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns. Magnetohydrodynamic has attracted the attention of a large number of scholars due to its diverse applications. Convective heat transfer through porous media has been a subject of great interest for the last three decades. Kim *et al.* [1] and Harris *et al.* [2] solved the problem of natural convection flow through porous medium past a plate. Raptis and Kafousias [3] analyzed the effects of magnetic field on steady free convection flow through a porous medium bounded by an infinite vertical plate. Raptis [4] investigated time varying two-dimensional natural convection flow of a viscous, incompressible and electrically conducting fluid past an infinite vertical porous plate embedded in a porous medium. Chamkha [5] studied MHD free convection from a vertical plate embedded in a thermally stratified porous medium. Chamkha [6] also investigated MHD free convection from vertical porous plate embedded in a porous medium with Hall effects.

Verma[7] studied the transient natural convection boundary layer flow, over a semi-infinite horizontal plate with the plate temperature oscillating with a non-zero mean, and a stationary free stream using matched asymptotic expansions and identifying the presence of four discrete oscillating layers at large distances from the leading edge of the plate. Singh *et al.* [8] have investigated the effects of surface temperature oscillation on transient two-dimensional free convective flow through a porous medium bounded by an infinite vertical plate using double asymptotic expansions in powers of the frequency parameter. Agrawal *et al.* [9] studied the influence of free convection and mass transfer on hydromagnetic flow past a vibrating infinite isothermal and constant heat flux vertical circular cylinder using Laplace transforms for both cooling (Grashof number greater than zero) and heating (Grashof number less than zero) of the cylinder). Takhar and Ram [10] considered Hall current and surface temperature oscillation effects on natural convection magnetohydrodynamic heat-generating flow. Soundalgekar *et al.* [11] obtained, exact solutions for heat transfer from an infinite vertical oscillating plate with foreign mass injection using the Laplace-transform technique with a linear temporal variation of plate temperature showing that Skin-friction increases with increasing frequency, Schmidt number and thermal Grashof number, but is reduced with a rise in species Grashof number.

Das *et al.* [12] have also investigated unsteady natural convection from a vertical surface with surface temperature oscillation effects. Merkin and Pop [13] investigated the unsteady free convection boundary-layer flow near the stagnation point of a two-dimensional cylindrical surface embedded in a fluid-saturated porous medium with surface temperature oscillation using both numerical and asymptotic solutions showing that external to the thin boundary layer on the surface a steady flow is induced for large times and rapid oscillations. Chaudhary and Jain [14] have discussed the combined effects of oscillating surface temperature and fluctuating surface velocity, on hydro magnetic convection with internal heat absorption. In all the above studies thermal radiation heat transfer was neglected, despite the important contribution this mode of heat transfer exerts in high temperature manufacturing processes, fire dynamics phenomena, rocket exhaust plumes etc. Anderson *et al*., (15) have studied the diffusion of a chemically reactive species from a linearly stretching sheet. Anjalidevi and Kandasamy (16) investigated the effect of a chemical reaction on the flow along a semi-infinite horizontal plate in the presence of heat transfer. Anjalidevi and Kandasamy (17) have studied the effect of a chemical reaction on the flow in the presence of heat transfer and magnetic field. Muthucumaraswamy and Ganesan (18) have analyzed the effect of a chemical reaction on the unsteady flow past an impulsively started semi-infinite vertical plate, which is subject to uniform heat flux. McLeod and Rajagopal (19) have investigated the uniqueness of the flow of a Navier Stokes fluid due to a linear stretching boundary. Raptis *et al*., (20) have studied the viscous flow over a non-linearly stretching sheet in the presence of a chemical reaction and magnetic field. In 1961, Sakiadis (21) who developed a numerical solution for the boundary layer flow field over a continuous solid surface moving with constant speed.

The aim of the *present* investigation is to study hydromagnetic free convection of an optically-thin gray gas from vertical flat plate subject to a surface temperature oscillation with significant thermal radiation, Chemical reaction and Heat source. The fluid is considered to be a gray, absorbing-emitting but non-scattering medium. The effects of the magnetic parameter (Hartmann number, M^2), free convection parameter (Grashof number, *Gr*), frequency of oscillation (ω) and the radiation parameter (K_1) on primary and secondary velocity distributions, Shear stresses, Nusselt number and Sherwood number at the plates are presented graphically and in tabular form.

MATHEMATICAL MODEL AND COMPLEX VARIABLE SOLUTIONS

We consider a two-dimensional free convection effects on unsteady MHD heat and mass transfer characteristics of a radiated vertical porous plat moving with constant velocity, U, in the presence of transverse magnetic field. The surface temperature of the plate oscillates with small amplitude about a non-uniform mean temperature and the concentration oscillation is also considered. The co-ordinate system is such that the x^*- *axis* is taken along the plate and y^* - *axis* is normal to the plate. All the fluid properties are considered to be constant except the influence of the density variation in the buoyancy term, according to the classical Boussinesq approximation. Since magnetic Reynolds number is very small for metallic liquid or partially ionized fluid the induced magnetic field produced by the electrically-conducting fluid is negligible. The Hall effects, the viscous dissipation and the joule heating terms are also neglected. The governing equation for this investigation is based on the balances of mass, linear momentum, energy, and species concentration. The equations can be written in Cartesian frame of reference, as follows.

$$\frac{\partial u^*}{\partial y^*} + \frac{\partial w^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} = \vartheta \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T^* - T^*_\infty) + g\beta_1(C^* - C^*_\infty) - \frac{\vartheta}{K^*}u^* - \frac{\sigma B_0^2}{\rho}u^* \tag{2}$$

$$\frac{\partial w^*}{\partial t^*} = \vartheta \frac{\partial^2 w^*}{\partial y^{*2}} - \frac{\vartheta}{K^*}w^* - \frac{\sigma B_0^2}{\rho}w^* \tag{3}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*} + Q_0(T^*_\infty - T^*) \tag{4}$$

$$\frac{\partial C^*}{\partial t^*} = D^* \frac{\partial^2 C^*}{\partial y^{*2}} - k_1^*(C^* - C^*_\infty) \tag{5}$$

Where $u^*$and w^* are x^*-direction and z^*-direction velocity components. ρ is the fluid density, μ is the viscosity, c_p is the specific heat at constant pressure, σ is the fluid electrical conductivity, β is the volumetric coefficient of the thermal expansion, β_1 is the volumetric coefficient of the concentration expansion, B_0 is the magnetic induction, K^* is the permeability of the porous medium, T^* is the dimensional temperature, D^* is the coefficient of chemical molecular diffusivity, C^* is the dimensional concentration, k is the thermal conductivity of the fluid, g is the acceleration due to gravity, q_r^* is the local radiative heat flux, k_1^* the reaction rate constant respectively.

The term Q_0 $(T - T_\infty)$ is assumed to be amount of heat generated or absorbed per unit volume Q_0 is a constant, which may take on either positive or negative values. When the wall temperature T^* exceeds the free stream temperature T^*_∞, the source term $Q_0 > 0$ and heat sink when $Q_0 < 0$.

The boundary conditions for the velocity, temperature, and concentration fields are given as follows

$$u^* = U,\ w^* = 0,\ \phi = T^* - T^*_\infty = \theta_w(x)(1 + \xi e^{iw^*t^*}),$$
$$\psi = C^* - C^*_\infty = C_w(1 + \xi e^{iw^*t^*}) \qquad y^* = 0$$
$$u^* \to 0,\ w^* \to 0,\ T^* \to T^*_\infty,\ C^* \to C^*_\infty \qquad y^* \to \infty \tag{6}$$

Where ϕ is designated as wall-free stream temperature difference, ψ is designated as free stream concentration difference, $\xi = \frac{U}{\vartheta}$ dimensionless velocity ratio and ω is the frequency of oscillation in the surface temperature of the plate.

The conditions (6) suggest solutions to equations (1) to (5) for the variables u^*, w^*, ϕ *and* ψ of the form.

$$u^* = u_0^* + \epsilon e^{iw^*t^*} u_1^*$$
$$w^* = w_0^* + \epsilon e^{iw^*t^*} w_1^*$$
$$\phi = T^* - T^*_\infty = \theta_w(x)(\theta_0^* + \xi e^{iw^*t^*}\theta_1^*) \tag{7}$$
$$\psi = C^* - C^*_\infty = C_w(c_0^* + \xi e^{iw^*t^*} c_1^*)$$

For the case of an optically-thin gray gas, the thermal radiation flux gradient may be

Expressed as follows (Siegel and Howell 1993):

$$-\frac{\partial q_r^*}{\partial y^*} = 4a\sigma^*\,(T_\infty^{*4} - T^{*4}) \tag{8}$$

And q_r^* is the radiative heat flux, *a* is the absorption coefficient of the fluid and σ^* is the Stefan-Boltzmann constant. We assume that the temperature differences within the flow are sufficiently small such that T^{*4} may be expressed as a *linear* function of the temperature. This is accomplished by expanding T^{*4} in a Taylor series about T_∞^{*4} and neglecting higher order terms, leading to:

$$T^{*4} = 4T_\infty^{*3}T^* - 3T_\infty^{*4} \tag{9}$$
$$\frac{\partial q_r^*}{\partial y^*} = 16a\sigma^* T_\infty^{*3}(T^* - T_\infty^*) \tag{10}$$

The components u_0^*, w_0^* represent the *steady* mean flow, θ_0^* represents temperature fields, c_0^* represents concentration fields and satisfy the following equations:

$$\frac{\partial u_0^*}{\partial y^*} + \frac{\partial w_0^*}{\partial y^*} = 0 \tag{11}$$

$$\vartheta \frac{\partial^2 u_0^*}{\partial y^{*2}} + g\beta\theta_w(x)\theta_0^* + g\beta_1 C_w c_0^* - \frac{\vartheta}{K^*}u_0^* - \frac{\sigma B_0^2}{\rho}u_0^* = 0 \tag{12}$$

$$\vartheta \frac{\partial^2 w_0^*}{\partial y^{*2}} - \frac{\vartheta}{K^*}w_0^* - \frac{\sigma B_0^2}{\rho}w_0^* = 0 \tag{13}$$

$$\frac{k}{\rho C_p}\frac{\partial^2 \theta_0^*}{\partial y^{*2}} - \frac{16a\sigma^* T_\infty^{*3}}{\rho C_p}\theta_0^* + Q_0\theta_0^* = 0 \tag{14}$$

$$D^*\frac{\partial^2 c_0^*}{\partial y^{*2}} - k_1^* c_0^* = 0 \tag{15}$$

The corresponding boundary conditions are

$$u_0^* = U,\ \ w_0^* = 0,\ \ T^* = T_w^*,\ C^* = C_w^* \qquad y^* = 0$$

$$u_0^* \to 0, w_0^* \to 0,\ \ T^* \to T_\infty^*,\ C^* \to C_\infty^* \qquad y^* \to \infty \tag{16}$$

The components u_1^*, w_1^* represent the un*steady* mean flow, θ_1^* represents temperature fields, c_1^* represents concentration fields and satisfy the following equations:

$$\frac{\partial u_1^*}{\partial y^*} + \frac{\partial w_1^*}{\partial y^*} = 0 \tag{17}$$

$$i\,\omega^* u_1^* = \vartheta\frac{\partial^2 u_1^*}{\partial y^{*2}} + g\beta\theta_w(x)\theta_1^* + g\beta_1 C_w c_1^* - \frac{\vartheta}{K^*}u_1^* - \frac{\sigma B_0^2}{\rho}u_1^* \tag{18}$$

$$i\,\omega^* w_1^* = \vartheta\frac{\partial^2 w_1^*}{\partial y^{*2}} - \frac{\vartheta}{K^*}w_1^* - \frac{\sigma B_0^2}{\rho}w_1^* \tag{19}$$

$$i\,\omega^*\theta_1^* = \frac{k}{\rho C_p}\frac{\partial^2 \theta_1^*}{\partial y^{*2}} - \frac{16a\sigma^* T_\infty^{*3}}{\rho C_p}\theta_1^* + Q_0\theta_1^* \tag{20}$$

$$i\,\omega^* c_1^* = D^*\frac{\partial^2 c_1^*}{\partial y^{*2}} - k_1^* c_1^* \tag{21}$$

The corresponding boundary conditions are

$$u_1^* = U,\ \ w_1^* = 0,\ T^* = T_w^*,\ C^* = C_w^* \qquad y^* = 0$$

$$u_1^* \to 0,\ w_1^* \to 0,\ T^* \to T_\infty^*,\ C^* \to C_\infty^* \qquad y^* \to \infty \tag{22}$$

We introduce following dimensionless quantities to normalize the flow mode:

$$\left.\begin{array}{c} u_0 = \frac{u_0^*}{U}, w_0 = \frac{w_0^*}{U},\ u_1 = \frac{u_1^* e}{U}, w_1 = \frac{w_1^* e}{U},\ C_0^* = \frac{c_0 \vartheta}{UL}, C_1^* = \frac{c_1 \vartheta}{ULe} \\ y = \frac{y^* U}{\vartheta},\ t = \frac{t^* U^2}{\vartheta}, \theta_0^* = \frac{\theta_0 \vartheta}{UL}, \theta_1^* = \frac{\theta_1 \vartheta}{ULe},\ \omega = \frac{\omega^* \vartheta}{U^2} \\ G_r = \frac{g\beta\vartheta^2\theta_w(x)}{U^4 L}, M = \frac{\sigma B_0^2 \vartheta}{\rho U^2},\ K_1 = \frac{16 a\sigma^* \vartheta^2 T_\infty^{*3}}{K^* U^2}, \\ S = \frac{Q_0 \rho \vartheta^2 C_p}{K^* U^2}, \gamma = \frac{k_1^* \vartheta}{U^2}, S_c = \frac{\vartheta}{D^*}, \\ P_r = \frac{\rho \vartheta C_p}{K^*}, G_m = \frac{g\beta_1 \vartheta^2 C_w}{U^4 L}, \theta_0 = \theta_1 = \frac{(T^* - T_\infty^*)}{T_w^* - T_\infty^*} \quad C_0 = C_1 = \frac{(C^* - C_\infty^*)}{C_w^* - C_\infty^*} \end{array}\right\} \tag{23}$$

Where G_r is the Grash of number, G_m is modified Grash of number, M is Magnetic parameter, K_1 is the thermal radiation-conduction number, γ is the chemical reaction parameter, θ_1 is dimensionless temperature and C_1 is dimensionless concentration. Using equation (23), the dimensionless form of the equations are given by

$$\frac{d^2 u_0}{dy^2} - M_1 u_0 + G_r \theta_0 + G_m C_0 = 0 \tag{24}$$

$$\frac{d^2 w_0}{dy^2} - M_1 w_0 = 0 \tag{25}$$

$$\frac{d^2 \theta_0}{dy^2} - (K_1 + S)\theta_0 = 0 \tag{26}$$

$$\frac{d^2 C_0}{dy^2} - \gamma S_c C_0 = 0 \tag{27}$$

$$\frac{d^2 u_1}{dy^2} - M_1 u_1 + G_r \theta_1 + G_m C_1 = 0 \tag{28}$$

$$\frac{d^2 w_1}{dy^2} - (i\omega + M_{1)} w_1 = 0 \tag{29}$$

$$\frac{d^2 \theta_1}{dy^2} - (i\omega P_r + K_1 + S)\theta_1 = 0 \tag{30}$$

$$\frac{d^2 C_1}{dy^2} - S_c(i\omega + \gamma) C_1 = 0 \tag{31}$$

The corresponding boundary conditions for steady mean flow are

$$u_0 = 1,\ w_0 = 0, \theta_0 = 1, C_0 = 1 \quad at\ y = 0$$

$$u_0 = 0, w_0 = 0, \theta_0 = 0,\ C_0 = 0, \quad at\ y \to \infty \tag{32}$$

The corresponding boundary conditions for unsteady mean flow are

$$u_1 = 1, w_1 = 0, \theta_1 = 1, C_1 = 1 \qquad at\ y = 0$$

$$u_1 = 0, w_1 = 0, \theta_1 = 0,\ C_1 = 0, \qquad at\ y \to \infty \tag{33}$$

Introducing the complex variable $= u_0 + iw_0,\ H = u_1 + iw_1$, eqs (24), (25), (28) and (29) implies

$$\frac{d^2F}{dy^2} - M_1F = -G_r\theta_0 - G_cC_0 \tag{34}$$

$$\frac{d^2H}{dy^2} - (i\omega + M_1)H - G_r\theta_1 - G_rC_1 \tag{35}$$

The corresponding boundary conditions are:

$$F = 1,\ \ \theta_0 = 1,\ \ C_0 = 1 \qquad at\ y = 0$$

$$F = 0\ , \theta_0 = 0,\ C_0 = 0, \qquad at\ y \to \infty \tag{36}$$

$$H = 1,\ \ \theta_1 = 1,\ \ C_1 = 1 \qquad at\ y = 0$$

$$H = 0\ , \theta_1 = 0,\ C_1 = 0, \qquad at\ y \to \infty \tag{37}$$

Solutions

Solving the above set of equations with respect to the corresponding boundary conditions, the following solutions are obtained.

$$\theta_0 = e^{-\sqrt{S_1}y} \tag{40}$$

$$C_0 = e^{-\sqrt{S_2}y} \tag{39}$$

$$\theta_1(y,t) = e^{-A_2y}cosB_2y - ie^{-A_2y}sinB_2 \tag{40}$$

$$C_1(y,t) = e^{-A_3y}cosB_3y - ie^{-A_3y}sinB_3y \tag{41}$$

Equation (34) subject to the boundary conditions (36) can be solved and solution for the steady mean flow is be expressed as

$$F = u_0(y) + iw_0(y)$$

$$where\ \ u_0(y) = m_3e^{-\sqrt{M_1}y} - m_2e^{-\sqrt{S_1}y} - m_1e^{-\sqrt{S_2}y} \tag{42}$$

$$w_0(y) = 0 \tag{43}$$

$$H = u_1(y,t) + iw_1(y,t)$$

$$u_1(y,t) = [m_9\cos B_1y - m_8SinB_1y]e^{-A_1y} - [m_7\cos B_2y - m_6SinB_2y]e^{-A_2y} - [m_5\cos B_3y - m_4SinB_3y]e^{-A_3y} \tag{44}$$

$$w_1(y,t) = [m_9\, SinB_1 y - m_8 CosB_1 y]e^{-A_1 y} + [m_7\, SinB_2 y + m_6 CosB_2 y]e^{-A_2 y} + [m_5\, SinB_3 y + m_4 CosB_3 y]e^{-A_3 y} \tag{45}$$

The functions θ_0 *and* θ_1 denote the temperature fields due to the main flow and cross flows, respectively.

Of interest in practical MHD plasma energy generator design are the dimensionless shear stresses at the plate, which may be defined for steady and unsteady mean flow, respectively, as follows:

$$\left(\frac{\partial F}{\partial y}\right)_{y=0} = -\sqrt{M_1}\left(1 + \frac{G_r}{S_1 - M_1} + \frac{G_r}{S_2 - M_1}\right) + \frac{G_r\sqrt{S_1}}{S_1 - M_1} + \frac{G_m\sqrt{S_2}}{S_2 - M_1} \tag{46}$$

$$\left(\frac{\partial H}{\partial y}\right)_{y=0} = -(A_1 + iB_1) + \frac{\alpha_1 - i\beta_1}{\alpha_1{}^2 + \beta_1{}^2} G_r[-(A_1 + iB_1) + (A_2 + iB_2)]$$

$$+ \frac{\alpha_2 - i\beta_2}{\alpha_2{}^2 + \beta_2{}^2} G_m[-(A_1 + iB_1) + (A_3 + iB_3)] \tag{47}$$

It is evident from equations (46) and (47) that the shear stress component due to the main flow for the steady mean flow given by equation (46) and the shear stress components due to main and cross flows given by equation (47) do not vanish at the plate. Inspection of these expressions also reveals that the shear stress component as defined by equation (46) due to steady mean flow is subjected to a non-periodic oscillation that depends on Hartmann number and radiation conduction parameter. In contrast to this, the shear stress components as computed in equation (47) due to the main and cross flows for an unsteady mean flow are subjected to periodic oscillation which is a function of not only Hartmann number and radiation-conduction parameter, but also the Prandtl number and the frequency of oscillation. The shear stress for the equation (46) will vanish at the plate (y=0) at a critical value of the free convection parameter i.e. Grash of number, defined by the conduction:

$$Gr_{Crit} = \sqrt{M_1}(\sqrt{S_1} + \sqrt{M_1}) \text{ And } G_m = 0 \tag{48}$$

The shear stress for equation (47) will vanish at the plate (y=0) When:

$$Gr_{crit} = (A_1 + iB_1)[(A_1 + A_2) + i((B_1 + B_2)] \text{ and } G_m = 0 \tag{49}$$

Also of interest in plasma MHD generator design is the dimensionless temperature gradient at the plate. This can be shown to take the form, for the unsteady main flow, as follows:

$$\left(\frac{\partial \theta_0}{\partial y}\right)_{y=0} = -\sqrt{S_1} = \sqrt{S + K_1} \tag{50}$$

For the unsteady cross flow the dimensionless temperature gradient at the plate (y=0) is:

$$\left(\frac{\partial \theta_1}{\partial y}\right)_{y=0} = -\sqrt{S_3} = -\sqrt{S_1 + i\omega P_r} \tag{51}$$

For the unsteady main flow and cross flow the dimensionless concentration gradient at the plate (y=0) is given by:

$$\left(\frac{\partial C_0}{\partial y}\right)_{y=0} = -\sqrt{S_2} = -\sqrt{vS_c} \tag{52}$$

$$(\frac{\partial C_1}{\partial y})_{y=0} = -\sqrt{S_4} = -\sqrt{S_2 + i\omega S_c} \tag{53}$$

RESULTS AND DISCUSSIONS

The selected computations for the velocity, temperature and concentration for different values of the material parameters have been presented in figures 1 - 20. Computations for the shear stresses, temperature gradient at the plate for steady and unsteady mean flow are provided in Tables 1 - 5. We note that in figures 1 - 6, only steady mean flow is simulated, for which there will be no surface temperature oscillation. Figures 7 - 14, correspond to the unsteady mean flow distribution due to the main flow (u_1) and figures 15 - 20 corresponds to the unsteady mean flow due to cross flow (w_1).

An increase in radiation conduction number K_1 has an adverse effect on the velocity due to steady mean flow (u_0) for all values of y, as shown in figure 1. K_1 represents the relative contribution of thermal radiation heat transfer to thermal conduction heat transfer. For $K_1 < 1$ thermal conduction exceeds thermal radiation and for $K_1 > 1$ this situation is reversed. For $K_1 = 1$ the contribution from both modes is equal. From figure 2, it is observed that, an increase in free convection parameters, Grash of number (Gr), increases the velocity due to steady mean flow. Similar effect has been observed from figure 3, in the case of modified Grash of number Gm. An increase in the magnetic parameter (M) has an inhibiting effect on steady mean flow as shown in figure 4. From figure 5, it is noticed that, as chemical reaction parameter (γ) increases, the velocity due to steady mean flow tends to decrease. An increase in Schmitt number (sc), has an adverse effect on the velocity due to steady mean flow (u_0) as shown in figure 6.

From figure 7, it is observed that, an increase in radiation conduction number K_1 decreases the unsteady mean flow (u_1). Whereas cross flow velocity (w_1) increases as K_1 increases (as shown in Figure 15). The effects of free convection parameters, Grash of number (Gr) and Modified Grash off number on unsteady mean flow are similar to for steady mean flow as shown in figures 8 and 9. An increase in the magnetic parameter M, has an adverse effect on the velocity due to unsteady mean flow as shown in figure 10. From figure 11, it is noticed that, as chemical reaction parameter (γ) increases, the velocity due to unsteady mean flow tends to decrease. From figure 12, it is observed that, as Prandtl number (Pr) increases, the velocity due to unsteady mean flow tends to decrease. The effect of oscillation frequency (ω) is computed, in figure 13, on the unsteady mean flow velocity component decreases as ω increases. From figure 14, it is observed that, an increase in Schmitt number (sc), decreases the velocity due to unsteady mean flow.

From figure 15, it is noticed that, an increase in Grash off number, decreases the velocity due to unsteady mean flow distribution due to cross flow (w_1), where as reverse effect is observed from figure 16 as modified Grash off number increases. From figure 17, unsteady mean cross flow velocity decreases in magnitude with a rise in magnetic parameter M. Unsteady mean flow distribution due to cross flow decreases as oscillation frequency (ω) increases as shown in figure 18. The effects of M, K_1, Gr, Gm, Sc, ω on share stress at the plate due to steady mean flow, unsteady main flow and unsteady cross flow are numerically shown in Table 1, Table 2 and Table 3 respectively. Also the effects of K_1, and S on the temperature gradient at the plate due to unsteady main flow are numerically shown in Table 4.

From Table 1 it is noticed that an increase in M, K_1 and Gm results in a decrease in share stress at the plate due to steady mean flow, while it increases as an increase in Gr and Sc. From Table 2, it is observed that an increase in M, K1,

Gr, Sc and ω leads to an increase in magnitude in the share stress at the plate due to unsteady main flow, while reverse effect is noticed for an increase in Gm. From Table 3, it is noticed that an increase in M and K_1 leads to an increase in magnitude in the share stress at the plate due to unsteady cross flow while reverse effect is noticed for an increase in Gr, Gm, Sc and ω. From Table 4, it is noticed that an increase in K_1 and S leads to an increase in the temperature gradient at the plate due to unsteady main flow.

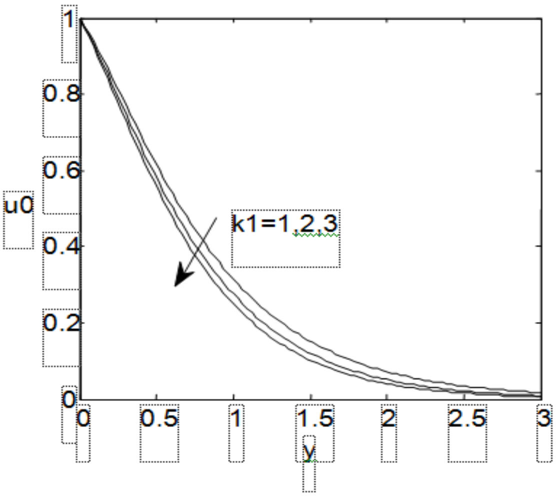

Figure 1: Velocity Distribution Due to a Steady Mean Flow (u_0) for Various Thermal Radiation-Conduction Numbers (K_1)

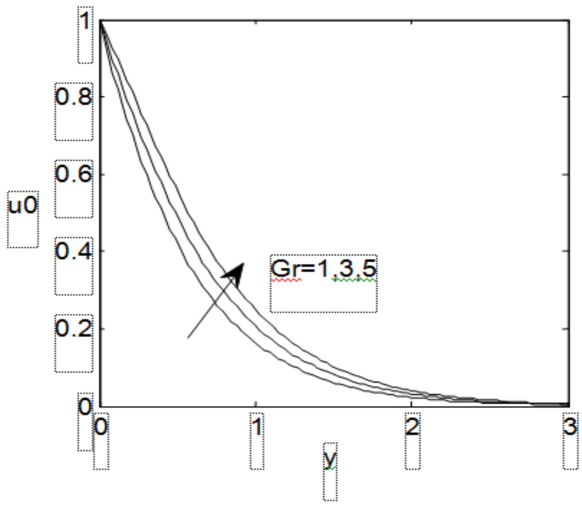

Figure 2: Velocity Distribution due to a Steady Mean Flow (u0) for Various Grashof Numbers (Gr)

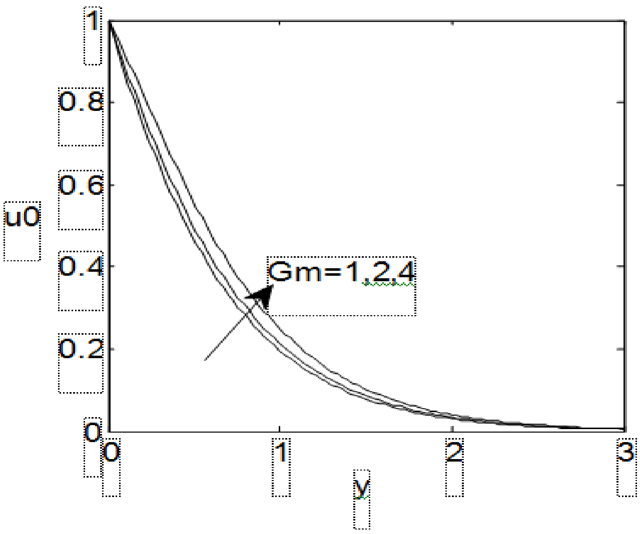

Figure 3: Velocity Distribution Due to a Steady Mean Flow (U0) for Various Modified Grash of Numbers (Gm)

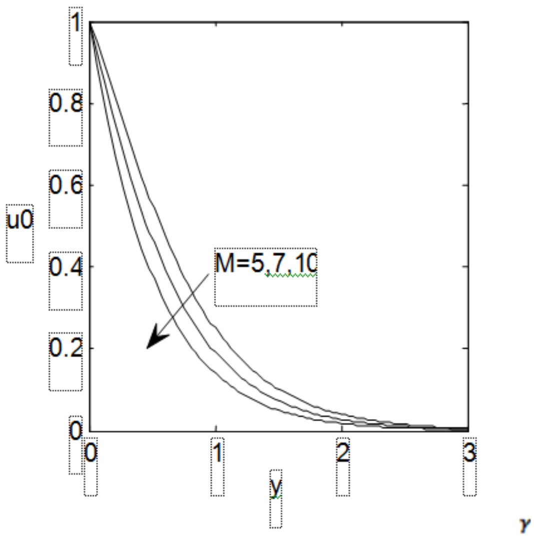

Figure 4: Velocity Distribution Due to a Steady Mean Flow (u0) for Various Values of Magnetic Parameter (M)

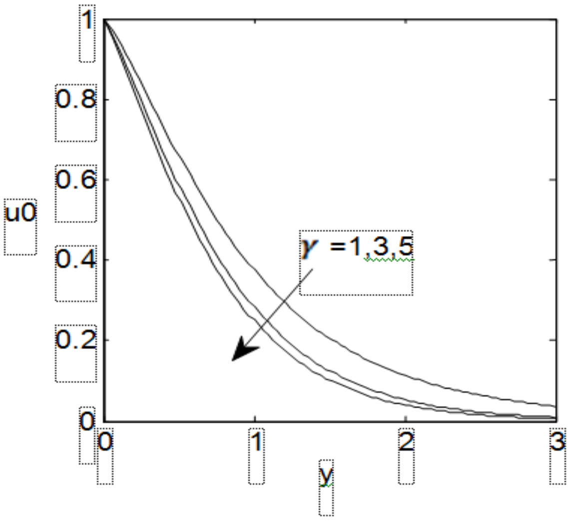

Figure 5: Velocity Distribution Due to a Steady Mean Flow (u0) for Various Values of Chemical Reaction (γ)

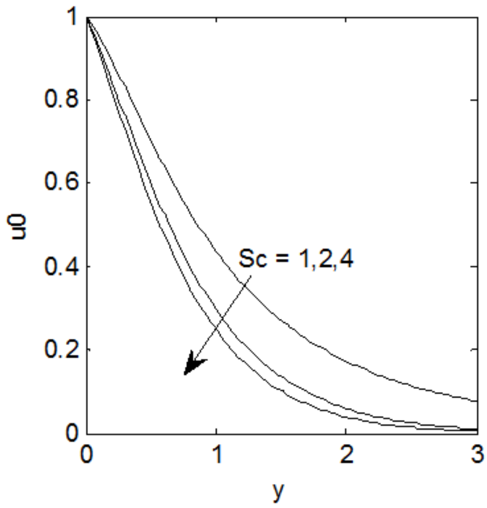

Figure 6: Velocity Distribution Due to a Steady Mean Flow (u0) For Various Values of Schmitt Number (Sc)

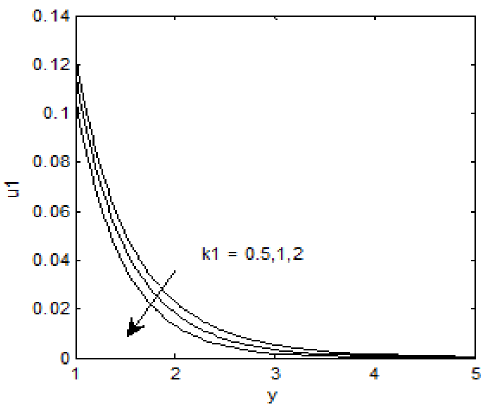

Figure 7: Unsteady Mean Flow distribution Due to Main Flow (u1) For Various Thermal Radiation-Conduction numbers (k1)

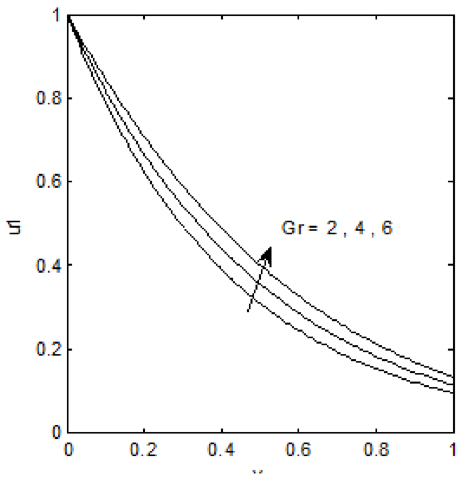

Figure 8: Unsteady Mean Flow Distribution Due to Main Flow (u1) For Various Values of Grashof numbers (Gr)

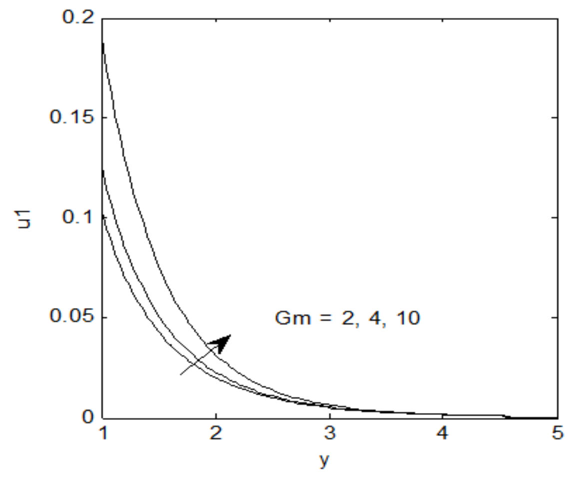

Figure 9: Unsteady Mean Flow Distribution Due to Main Flow (u1) For Various Values of Modified Grashof Numbers (Gm).

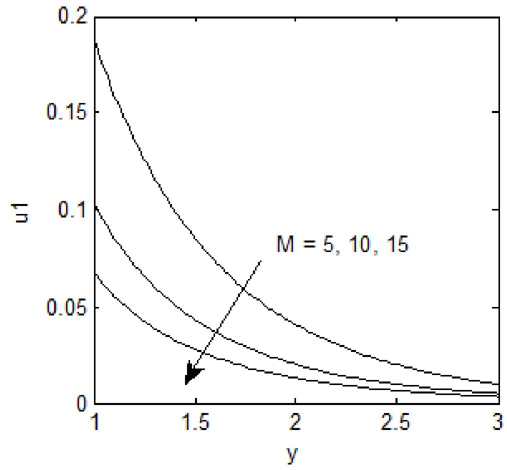

Figure 10: Unsteady Mean Flow Distribution Due to Main Flow (u1)for Various Values of Magnetic Parameter (M)

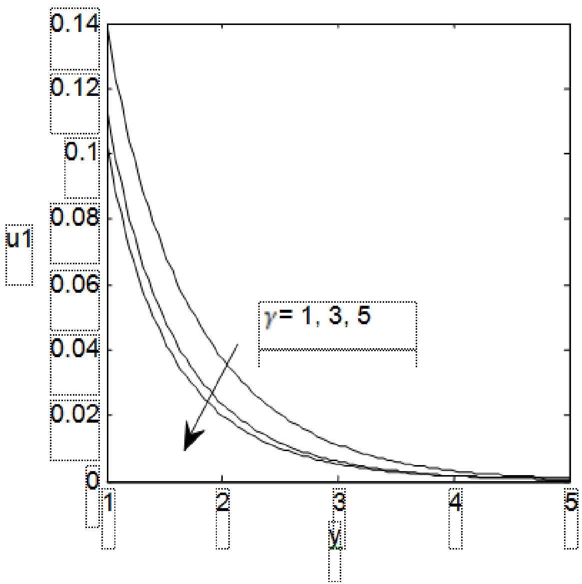

Figure 11: Unsteady Mean Flow Distribution Due to Main Flow (u1) For Various Values of Chemical Reaction (γ).

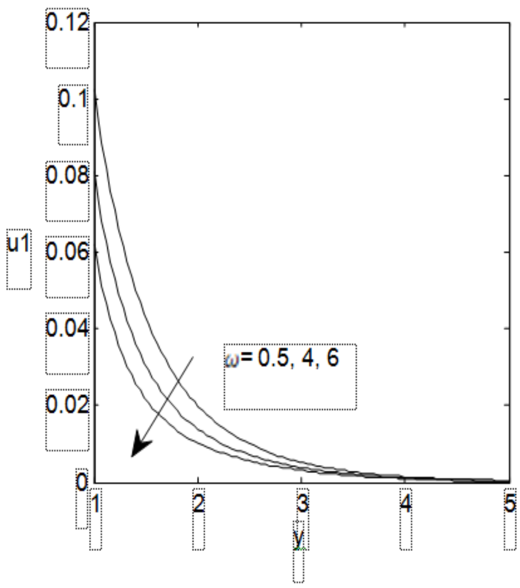

Figure 12: Unsteady Mean Flow Distribution Due to Main Flow (u1) For Various Values of Frequency of Oscillation (ω).

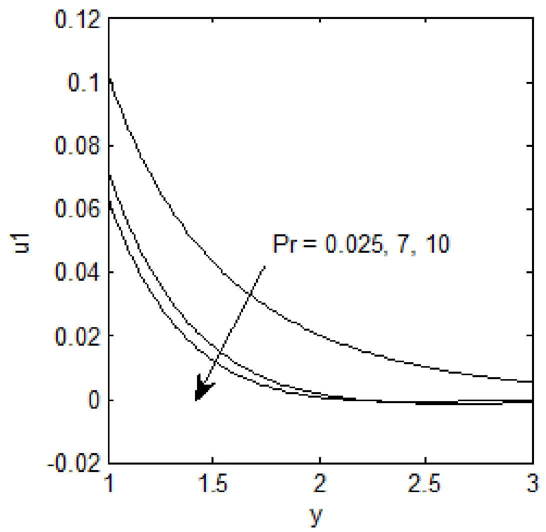

Figure 13: Unsteady Mean Flow Distribution Due to Main Flow (u1) For Various Values of Prandlt Numbers (Pr)

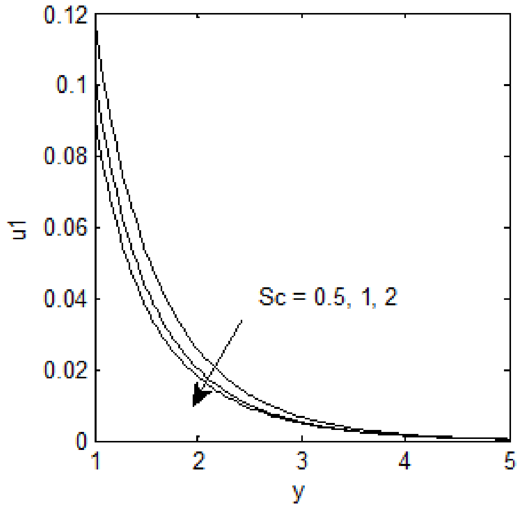

Figure 14: Unsteady Mean Flow Distribution Due to Main Flow (u1) For Various Values of Schmitt Number (Sc)

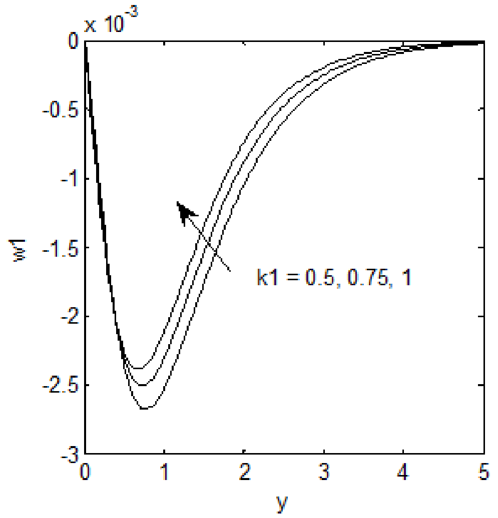

Figure 15: Unsteady Mean Flow Distribution due to Cross Flow (w1) For Various Thermal Radiation-Conduction Numbers (k1)

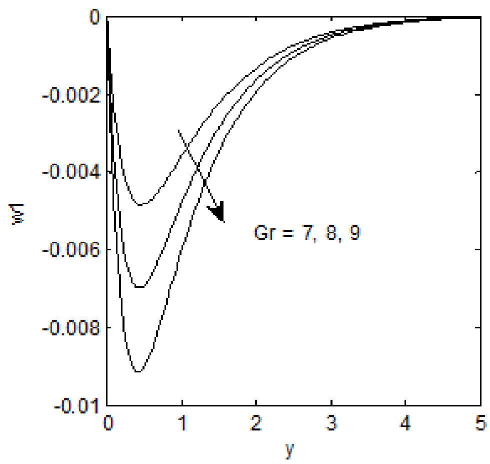

Figure 16: Unsteady Mean Flow Distribution Due to Cross Flow (w1) For various Values of Grash of Numbers (Gr)

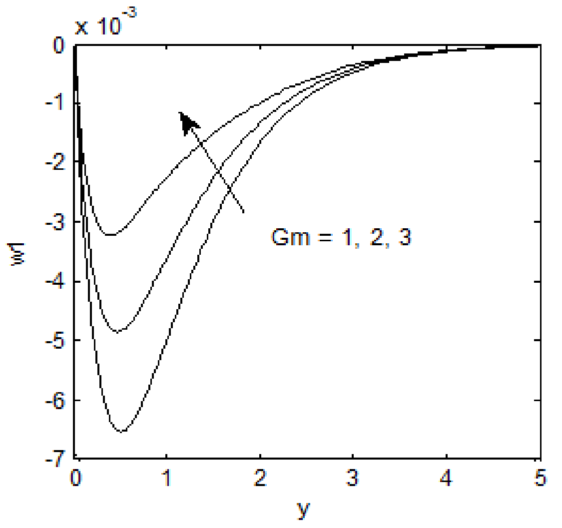

Figure 17: Unsteady Mean Flow Distribution due to Cross Flow (w1) For Various Values Modified of Grash of Numbers (Gm)

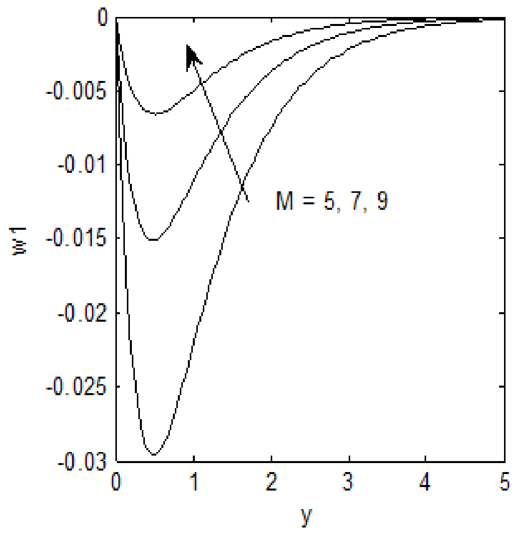

Figure 18: Unsteady Mean Flow Distribution Due to Cross Flow (w1) For Various Values of Magnetic Parameter (M)

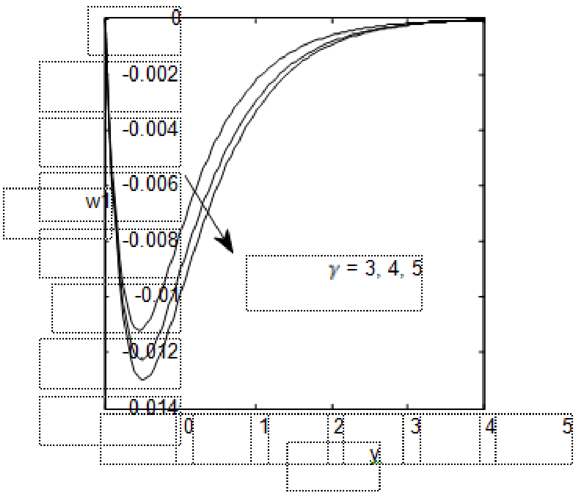

Figure 19: Unsteady Mean Flow Distribution Due to Cross Flow (w1) For Various Values of Chemical Reaction (γ)

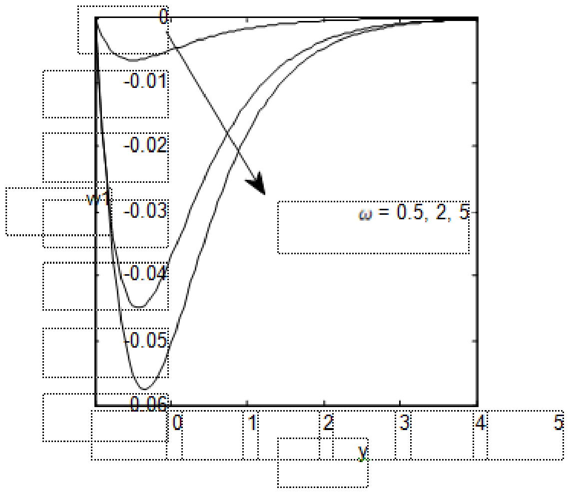

Figure 20: Unsteady Mean Flow Distribution due to Cross Flow (w1) For Various Values of Frequency of Oscillation (ω)

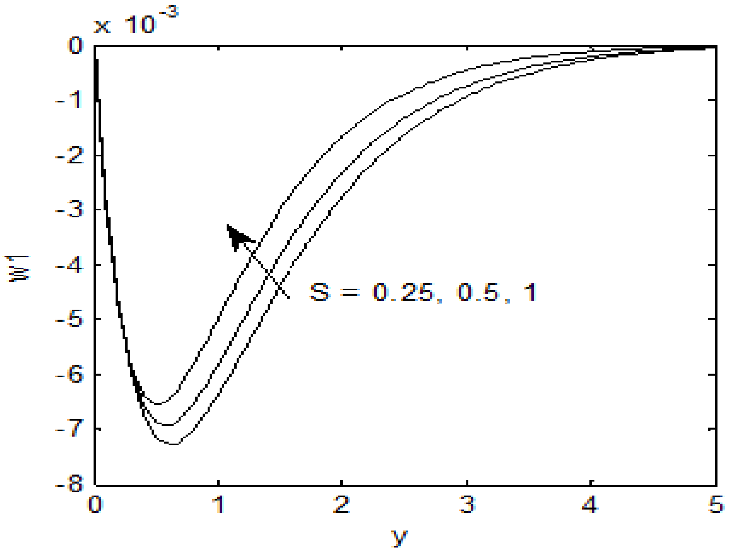

Figure 21: Unsteady Mean Flow Distribution Due to Cross Flow (w1) For various values of S

Table 1: Shear Stress at the Plate Due to Steady Mean Flow (u_0)

M	K_1	Gr	Gm	Sc	$\left(\frac{du_0}{dy}\right)_{y=0}$
5.00	0.50	7.00	4.00	0.50	1.8730
7.00	0.50	7.00	4.00	0.50	0.9147
5.00	**1.00**	7.00	4.00	0.50	1.7886
5.00	0.50	**9.00**	4.00	0.50	3.5656
5.00	0.50	7.00	**5.00**	0.50	1.5216
5.00	0.50	7.00	4.00	**1.00**	3.9508

Table 2: Shear Stress at the Plate Due to Unsteady Mean Flow Due to Main Flow (u_1)

M	K_1	Gr	Gm	Sc	ω	$\left(\frac{du_1}{dy}\right)_{y=0}$
5.00	0.50	7.00	4.00	0.50	0.50	-0.1807
7.00	0.50	7.00	4.00	0.50	0.50	-0.7068
5.00	**1.00**	7.00	4.00	0.50	0.50	-0.2019
5.00	0.50	**9.00**	4.00	0.50	0.50	0.2545
5.00	0.50	7.00	**5.00**	0.50	0.50	0.0553
5.00	0.50	7.00	4.00	**1.00**	0.50	-0.3072
5.00	0.50	7.00	4.00	0.50	**2.00**	-0.7630

Table 3: Shear Stress at the Plate Due to Unsteady Mean Flow Due To Cross Flow (w_1)

M	K_1	Gr	Gm	Sc	Ω	$\left(\frac{dw_1}{dy}\right)_{y=0}$
5.00	0.50	7.00	4.00	0.50	0.50	-0.6958
7.00	0.50	7.00	4.00	0.50	0.50	-0.5313

Table 3: Contd.,

5.00	**1.00**	7.00	4.00	0.50	0.50	-0.6737
5.00	0.50	**9.00**	4.00	0.50	0.50	-0.8677
5.00	0.50	7.00	**5.00**	0.50	0.50	-0.7430
5.00	0.50	7.00	4.00	**1.00**	0.50	-0.7011
5.00	0.50	7.00	4.00	0.50	**2.00**	-1.0356

Table 4: Temperature Gradient at the Plate Due to Steady Mean Flow (θ_0)

K1	S	$\left(\frac{d\theta_0}{dy}\right)_{y=0}$
0.50	1.00	1.2247
1.00	1.00	1.4142
0.50	**2.00**	1.7321

CONCLUSIONS

Exact solutions have been derived using complex variables for the transient magneto hydrodynamic (MHD) convection flow of an electrically-conducting, Newtonian, optically-thin fluid from a flat plate with thermal radiation, chemical reaction, heat source and surface temperature oscillation effects. Our analysis has shown that:

- Steady mean flow velocity, u_0, is decreased with increasing of thermal radiation (K_1),magnetic parameter (M), chemical reaction (γ),Heat source parameter (S) and Schmitt number (Sc) ,while it is increased with the increase of Grashof number (Gr) and modified Grashof number (Gm).
- Unsteady main flow velocity (u_1) is decreased with increasing of thermal radiation (K_1),magnetic parameter (M), chemical reaction (γ),Heat source parameter (S), frequency of oscillation (ω) and Schmitt number (Sc) ,while it is increased with the increase of Grashof number (Gr) and modified Grashof number (Gm).
- Conversely cross flow velocity (w_1) is increased with increasing of thermal radiation (K_1),magnetic parameter (M), Heat source parameter (S) and modified Grashof number (Gm),while it is decreased with the increase of Grashof number (Gr), chemical reaction parameter (γ) and frequency of oscillation (ω).
- An increase in M, radiation-conduction parameter K_1 *and* modified Grashof number (Gm) causes a decrease in the shear stress at the plate due to a steady mean flow (u_0).
- An increase in frequency of oscillation (ω), magnetic parameter (M), radiation-conduction parameter K_1 , Grashof number (Gr), Schmitt number (Sc) *and* modified Grashof number (Gm) increases shear stress at the plate due to unsteady mean flow of the main flow (u_1).
- Shear stress at the plate with unsteady mean flow due to cross flow (w_1) also decreases with an increase of magnetic parameter (M) and radiation-conduction parameter K_1 , but it is considerably increased with increase of Grashof number (Gr), Schmitt number (Sc),modified Grashof number (Gm) *and* frequency of oscillation (ω).
- An increase in thermal radiation-conduction parameter (K_1) and Heat Source parameter (S) increases the temperature gradient at the plate due to unsteady mean flow (θ_0) .

- In the absence of chemical reaction, concentration and heat source parameters, these results are in good agreement with the results of O. Anwar Beg and S. K. Ghosh [26].

APPENDICES

$$M_1 = M + \frac{1}{k},\ S_1 = S + k_1,\ S_2 = \gamma S_c,\ \ S_3 = S_1 + i\omega P_r,\ S_4 = S_2 + i\omega S_c$$

$$\alpha_1 = (S_1 - M_1),\ \beta_1 = \omega(P_r - 1),\ \alpha_2 = (S_2 - M_1),\ \beta_2 = \omega(S_c - 1),$$

$$A_1 = \frac{1}{\sqrt{2}}\left[\left(M_1^{\,2} + \omega^2\right)^{1/2} + M_1\right]^{1/2},\ B_1 = \frac{1}{\sqrt{2}}\left[\left(M_1^{\,2} + \omega^2\right)^{1/2} - M_1\right]^{1/2}$$

$$A_2 = \frac{1}{\sqrt{2}}\left[\left(S_1^{\,2} + \omega^2 P_r^{\,2}\right)^{1/2} + S_1\right]^{1/2},\ B_2 = \frac{1}{\sqrt{2}}\left[\left(S_1^{\,2} + \omega^2 P_r^{\,2}\right)^{1/2} - S_1\right]^{1/2}$$

$$A_3 = \frac{1}{\sqrt{2}}\left[\left(S_2^{\,2} + \omega^2 S_c^{\,2}\right)^{1/2} + S_2\right]^{1/2},\ B_3 = \frac{1}{\sqrt{2}}\left[\left(S_2^{\,2} + \omega^2 S_c^{\,2}\right)^{1/2} - S_2\right]^{1/2},\ m_1 = \frac{G_m}{S_2 - M_1}$$

$$m_2 = \frac{G_r}{S_1 - M_1},\ m_3 = (1 + m_1 + m_2),\ m_4 = \frac{G_m \beta_2}{\alpha_2^{\,2} + \beta_2^{\,2}},\ m_5 = \frac{G_m \alpha_2}{\alpha_2^{\,2} + \beta_2^{\,2}},\ m_6 = \frac{G_r \beta_1}{\alpha_1^{\,2} + \beta_1^{\,2}},$$

$$m_7 = \frac{G_r \alpha_1}{\alpha_1^{\,2} + \beta_1^{\,2}},\ m_8 = [C_6 + C_4],\ \ m_9 = [1 + C_7 + C_5]$$

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